AD-A174 429 TWO NEW SERIES OF SEARCH DESIGNS FOR 3(M) FACTORIAL EXPERIMENTS(U) CALIFORNIA UNIV RIVERSIDE DEPT OF STATISTICS S GHOSH ET AL 11 JUL 86 TR-144 1/1

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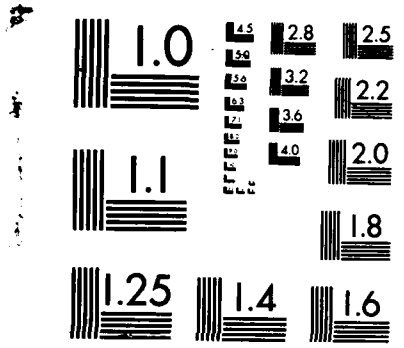

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Technical Report No. 144

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July 1986

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TWO NEW SERIES OF SEARCH DESIGNS FOR 3^m FACTORIAL EXPERIMENTS

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0. Summary

In this paper two new series of search designs with very small number of treatments are presented for 3^m factorial experiments. The first series of designs can search one nonzero two factor interaction and estimate it along with the general mean and the main effects. The second series of designs can search one nonzero three factor interaction and estimate it along with the two factor and lower order interactions.

Short Running Title: Search Designs for 3^m Factorials

AMS 1970 Subject Classifications: Primary and Secondary: 62K15

Keywords and Phrases: Estimability, Interactions, Main Effects, Resolution III and V Plans, Search Designs.

*The work of the first author is sponsored by the Air Force Office of Scientific Research under Grant AFOSR-86-0048.

**The work of the second author is supported by a fellowship from the University of California, Riverside.

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1. Introduction

Consider the linear model

$$E(\underline{y}) = X_1 \underline{\beta}_1 + X_2 \underline{\beta}_2, \tag{1}$$

$$V(\underline{y}) = \sigma^2 I, \tag{2}$$

where $\underline{y}$ (Nx1) is a vector of observations and for i = 1,2, X_i ($Nx\nu_i$) are known matrices, $\underline{\beta}_i$(ν_ix1) are vectors of fixed parameters and σ^2 is a constant which may or may not be known. Moreover, $\underline{\beta}_1$ is completely unknown, but we have partial information about $\underline{\beta}_2$. We know that at most p elements of $\underline{\beta}_2$ are nonzero and the remaining elements are negligible, where p is a nonnegative integer which may or may not be known. In this paper we assume p is known to be 1. However, we do not know exactly which element of $\underline{\beta}_2$ is nonzero. The problem is to search the nonzero element of $\underline{\beta}_2$ and draw inference on it in addition to the elements of $\underline{\beta}_1$. Such models are called search linear models and were introduced in Srivastava (1975). We want X_1 and X_2 to be such that the above problem can be resolved; the underlying design corresponding to X_1 and X_2 is called a search design.

In a 3^m factorial experiment the treatments are denoted by $(a_1,\ldots a_m)$, $a_i = 0,1,2$; the factorial effects are deonoted by $F_1^{c_1}\ldots F_m^{c_m}$ $c_i = 0,1,2$; the observation corresponding to the treatment $(a_1,\ldots,a_m)$ is denoted by $y(a_1,\ldots,a_m)$. The expectation form of the model is

$$E(y(a_1,\ldots,a_m)) = \Sigma\, b_1\ldots b_m F_1^{c_1}\ldots F_m^{c_m}, \tag{3}$$

where the values of b_i depend on a_i and c_i and are given in Table 1.

Table 1
The values of b_i

a_i \ c_i	0	1	2
0	1	-1	1
1	1	0	-2
2	1	1	1

We now consider the following two situations:

S1: The vector $\underline{\beta}_1$ consists of the general mean and the main effects and the vector $\underline{\beta}_2$ consists of the 2-factor interactions. The 3-factor and higher order interactions are assumed to be zero. We assume $m \geq 3$.

S2: The vector $\underline{\beta}_1$ consists of the general mean, the main effects and the 2-factor interactions and the vector $\underline{\beta}_2$ consists of the 3-factor interactions. The 4-factor and higher order interactions are assumed to be zero. We assume $m \geq 4$.

For given N treatments, we can write the equation (3) in the form of the equation (1) for both S1 and S2. Let D_1 be a design with (1+2m) treatments as the treatment with all factors at level 2, treatments with the ith (i = 1,...,m) factor at levels of 0 and 1 and the other factors at level 2. We know that under D_1 we can estimate the

elements of $\underline{\beta}_1$ in S1 with the assumpution that $\underline{\beta}_2 = \underline{0}$ (i.e., a Resolution III plan). Let D_2 be a design with $\left(1+2m+4\binom{m}{2}\right)$ treatments as (1+2m) treatments in D_1, treatments with the (i,j)th factors (i,j=1,..,m,i < j) at levels (0,0), (0,1), (1,0), (1,1) and the other factors at level 2. We know that under D_2 we can estimate the elements of $\underline{\beta}_1$ in S2 with the assumption that $\underline{\beta}_2 = \underline{0}$ (i.e., a Resolution V plan). Let D_3 be a design with m treatments as treatments with the ith (i=1,...,m) factor at level 0 and the other factors at level 1. We prove that the design $D^{(1)}$ consisting of treatments in D_1 and D_3 and the design $D^{(2)}$ consisting of the treatments in D_2 and D_3 are in fact search designs in S1 and S2, respectively.

In all Taguchi design methods, See Taguchi and Wu (1985), popular in statistical quality control experimentations, the higher order interactions (2-factor and higher order in most plans) are assumed to be zero. A few of those higher order interactions may have significant effect on the optimal experimental condition. The use of search designs may be a potential tool in improving upon the Taguchi design methods.

There is a vast literature available in the construction of search designs for 2^m factorial experiments. Near minimal resolution IV plan which permit search and estimation of three or fewer nonzero two factor interactions for p^m factorial experiments are available in Anderson and Thomas (1980) under the assumption that 3-factor and higher order interactions are all zero. Chatterjee and Mukherjee

(1986) presented search design for $s^r \times w^{(m-r)}$ where s and w are any positive integers, under the assumption that 3-factor and higher order interactions are all zero. Our $D^{(2)}$ in S2 is therefore totally new and there is no other competitor in literature. Our design $D^{(1)}$ in S1 is although new but has competitors in Chatterjee and Mukherjee (1986) and in Anderson and Thomas (1980). However, the design $D^{(1)}$ has an edge over designs in those papers in terms of the smaller number of treatments. This can be seen by taking examples 4.5 and 4.6 in Chatterjee and Mukherjee with $r = m$ and $s = 3$ and comparing with the number of treatments (1+3m) in $D^{(1)}$. In S1, the design in the example 4.5 in Chatterjee and Mukherjee has (1+6m) treatments with 3m more treatments than in $D^{(1)}$. In example 4.6 (m=3), Chatterjee and Mukherjee has 8 more treatments than in $D^{(1)}$. Indeed, both Chatterjee and Mukherjee, Anderson and Thomas designs can be used in factorial experiments other than 3^m.

2. Preliminary Results

We first introduce the following notations.

$y_{\alpha ij}^{\beta\gamma}$ = The observation corresponding to the treatment with levels of all factors except i and j are α, the level of the ith factor is β and the level of the jth factor is γ.

$y_{\alpha i}^{\beta}$ = The observation corresponding to the treatment with levels of all factors except i are α and the level of the ith factor is β.

y_α = The observation corresponding to the treatment with levels of all factors are α.

$S(c_i=u)$ = Sum of factorial effects $F_1^{c_1}\ldots F_m^{c_m}$ with $c_i=u$.

$S(c_i=u,c_j=v)$ = Sum of factorial effects $F_1^{c_1}\ldots F_m^{c_m}$ with $c_i=u$ and $c_j=v$.

We now present the minimum variance unbiased estimators (MVUE) of $S(c_i=u)$ and $S(c_i=u,c_j=v)$ under (1) and (2) with $\underline{\beta}_2=\underline{0}$ for both designs D_1 and D_2.

Table 2

MVUE's of $S(c_i=u)$ and $S(c_i=u,c_j=v)$ for D_1 and D_2

Design	Parameter	MVUE
D_1	$2\ S(c_i=1)$	$y_2-y_{2i}^0$
	$6\ S(c_i=2)$	$y_2-2y_{2i}^1+y_{2i}^0$
D_2	$4\ S(c_i=1,c_j=1)$	$y_2-y_{2i}^0-y_{2j}^0+y_{2ij}^{00}$
	$12\ S(c_i=1,c_j=2)$	$(y_2-y_{2i}^0)+(y_{2j}^0-y_{2ij}^{00})-2(y_{2j}^1-2y_{2ij}^{01})$
	$12\ S(c_i=2,c_j=1)$	$(y_2-y_{2j}^0)+(y_{2i}^0-y_{2ij}^{00})-2(y_{2i}^1-y_{2ij}^{10})$
	$36\ S(c_i=2,c_j=2)$	$(y_2+y_{2i}^0-2y_{2i}^1)+(y_{2j}^0-2y_{2ij}^{10}+y_{2ij}^{00})$ $-2(y_{2j}^1-2y_{2ij}^{11}+y_{2ij}^{01})$

The requirement on X_1 and X_2 for a design to be a search design (see, Srivastava (1975)) is that for $\binom{\nu_2}{2}$models

$$E(\underline{y}) = X_1\underline{\beta}_1 + X_2^{(i)}\,\underline{\beta}_2^{(i)},\ i=1,\ldots,\binom{\nu_2}{2}, \tag{4}$$

where $X_2^{(i)}$ (Nx2) is a submatrix of X_2 and $\underline{\beta}_2^{(i)}$ is a (2x1) subvector of $\underline{\beta}_2$, the parameters $\underline{\beta}_1$ and $\underline{\beta}_2^{(i)}$ are unbiasedly estimable. Both of $D^{(1)}$ and $D^{(2)}$ consist of two component designs namely (D_1,D_3) and (D_2,D_3). For u=1,2,3, we denote the observations corresponding to D_u by $\underline{y}_u$ and write (4) as

$$E(\underline{y}_u) = X_{u1}\underline{\beta}_1+X_{u2}^{(i)}\underline{\beta}_2^{(i)},\ i=1,\ldots,\binom{\nu_2}{2}. \tag{5}$$

For u=1,2, we have

$$\text{Rank } X_{u1} = \nu_1, \tag{6}$$

$$E[-X_{31}X_{u1}^{-1}\underline{y}_u+\underline{y}_3] = [X_{32}^{(i)}-X_{31}X_{u1}^{-1}X_{u2}^{(i)}]\underline{\beta}_2^{(i)}. \tag{7}$$

We denote

$$W_u^{(i)} = [X_{32}^{(i)}-X_{31}X_{u1}^{-1}X_{u2}^{(i)}],\ u=1,2,i=1,\ldots,\binom{\nu_2}{2}.$$

For u=1,2, the requirement for the design $D^{(u)}$ to be a search design is that

$$\text{Rank } W_u^{(i)} = \text{Rank } W_u^{(i)\prime}W_u^{(i)} = 2,\ i=1,\ldots,\binom{\nu_2}{2}.$$

It is to be noted that columns of $W_u^{(i)}$ (rows and columns of $W_u^{(i)\prime}W_u^{(i)}$)

correspond to a pair of elements of $\underline{\beta}_2$. Since both designs $D^{(1)}$ and $D^{(2)}$ are balanced arrays of full strength, it can be seen that for two pairs of elements of $\underline{\beta}_2$ isomorphic under the symmetric group of permutations of degree m on the set $\{1,2,\ldots,m\}$, the $W_u^{(i)}$ matrices are identical except for permutations of rows and columns. We now present in Table 3 and 4 the nonisomorphic pairs of elements in $\underline{\beta}_2$ for S1 and S2. We note that 2-factor interactions can be any of 3 types F_iF_j, $F_i^2F_j$, $F_i^2F_j^2$ for different values of i,j without assuming the restriction $i < j$. Similarly 3-factor interactions can be any of 4 types $F_iF_jF_k$, $F_i^2F_jF_k$, $F_i^2F_j^2F_k$, $F_i^2F_j^2F_k^2$ for different values of i,j,k without assuming the restriction $i < j < k$.

Table 3

The nonisomorphic pairs of elements in $\underline{\beta}_2$ for S1

(F_iF_j, F_iF_u)	$(F_iF_j, F_i^2F_u^2)$	$(F_i^2F_j, F_u^2F_v)$
(F_iF_j, F_uF_v)	$(F_iF_j, F_u^2F_v^2)$	$(F_i^2F_j, F_i^2F_j^2)$
$(F_iF_j, F_i^2F_j)$	$(F_i^2F_j, F_i^2F_u)$	$(F_i^2F_j, F_i^2F_u^2)$
$(F_iF_j, F_i^2F_u)$	$(F_i^2F_j, F_j^2F_i)$	$(F_i^2F_j, F_j^2F_u^2)$
$(F_iF_j, F_u^2F_i)$	$(F_i^2F_j, F_j^2F_u)$	$(F_i^2F_j, F_u^2F_v^2)$
$(F_iF_j, F_u^2F_v)$	$(F_i^2F_j, F_u^2F_i)$	$(F_i^2F_j^2, F_i^2F_u^2)$
$(F_iF_j, F_i^2F_j^2)$	$(F_i^2F_j, F_u^2F_j)$	$(F_i^2F_j^2, F_u^2F_v^2)$

Table 4

The nonisomorphic pairs of elements in $\underline{\beta_2}$ for S2

$(F_iF_jF_k,F_iF_jF_u)$	$(F_iF_jF_k,F_i^2F_u^2F_v^2)$	$(F_i^2F_jF_k,F_j^2F_k^2F_i)$	$(F_i^2F_j^2F_k,F_i^2F_u^2F_j)$
$(F_iF_jF_k,F_iF_uF_v)$	$(F_iF_jF_k,F_u^2F_v^2F_w^2)$	$(F_i^2F_jF_k,F_j^2F_k^2F_u)$	$(F_i^2F_j^2F_k,F_i^2F_u^2F_k)$
$(F_iF_jF_k,F_uF_vF_w)$	$(F_i^2F_jF_k,F_i^2F_jF_u)$	$(F_i^2F_jF_k,F_j^2F_u^2F_k)$	$(F_i^2F_j^2F_k,F_i^2F_u^2F_v)$
$(F_iF_jF_k,F_i^2F_jF_k)$	$(F_i^2F_jF_k,F_i^2F_uF_v)$	$(F_i^2F_jF_k,F_j^2F_u^2F_i)$	$(F_i^2F_j^2F_k,F_u^2F_k^2F_j)$
$(F_iF_jF_k,F_i^2F_jF_u)$	$(F_i^2F_jF_k,F_j^2F_iF_k)$	$(F_i^2F_jF_k,F_j^2F_u^2F_v)$	$(F_i^2F_j^2F_k,F_u^2F_k^2F_v)$
$(F_iF_jF_k,F_i^2F_uF_v)$	$(F_i^2F_jF_k,F_j^2F_iF_u)$	$(F_i^2F_jF_k,F_u^2F_v^2F_k)$	$(F_i^2F_j^2F_k,F_u^2F_v^2F_j)$
$(F_iF_jF_k,F_u^2F_jF_k)$	$(F_i^2F_jF_k,F_j^2F_uF_k)$	$(F_i^2F_jF_k,F_u^2F_v^2F_i)$	$(F_i^2F_j^2F_k,F_u^2F_v^2F_k)$
$(F_iF_jF_k,F_u^2F_jF_v)$	$(F_i^2F_jF_k,F_j^2F_uF_v)$	$(F_i^2F_jF_k,F_u^2F_v^2F_w)$	$(F_i^2F_j^2F_k,F_u^2F_v^2F_w)$
$(F_iF_jF_k,F_u^2F_vF_w)$	$(F_i^2F_jF_k,F_u^2F_jF_k)$	$(F_i^2F_jF_k,F_i^2F_j^2F_k^2)$	$(F_i^2F_j^2F_k,F_i^2F_j^2F_k^2)$
$(F_iF_jF_k,F_i^2F_j^2F_k)$	$(F_i^2F_jF_k,F_u^2F_iF_k)$	$(F_i^2F_jF_k,F_i^2F_j^2F_u^2)$	$(F_i^2F_j^2F_k,F_i^2F_j^2F_u^2)$
$(F_iF_jF_k,F_i^2F_j^2F_u)$	$(F_i^2F_jF_k,F_u^2F_iF_v)$	$(F_i^2F_jF_k,F_i^2F_u^2F_v^2)$	$(F_i^2F_j^2F_k,F_i^2F_k^2F_u^2)$
$(F_iF_jF_k,F_i^2F_u^2F_k)$	$(F_i^2F_jF_k,F_u^2F_vF_k)$	$(F_i^2F_jF_k,F_u^2F_i^2F_k^2)$	$(F_i^2F_j^2F_k,F_i^2F_u^2F_v^2)$
$(F_iF_jF_k,F_i^2F_u^2F_v)$	$(F_i^2F_jF_k,F_u^2F_vF_w)$	$(F_i^2F_jF_k,F_u^2F_v^2F_k^2)$	$(F_i^2F_j^2F_k,F_u^2F_k^2F_v^2)$
$(F_iF_jF_k,F_u^2F_v^2F_k)$	$(F_i^2F_jF_k,F_i^2F_j^2F_k)$	$(F_i^2F_jF_k,F_u^2F_v^2F_w^2)$	$(F_i^2F_j^2F_k,F_u^2F_v^2F_w^2)$
$(F_iF_jF_k,F_u^2F_v^2F_w)$	$(F_i^2F_jF_k,F_i^2F_j^2F_u)$	$(F_i^2F_j^2F_k,F_i^2F_j^2F_u)$	$(F_i^2F_j^2F_k^2,F_i^2F_j^2F_u^2)$
$(F_iF_jF_k,F_i^2F_j^2F_k^2)$	$(F_i^2F_jF_k,F_i^2F_u^2F_k)$	$(F_i^2F_j^2F_k,F_i^2F_k^2F_j)$	$(F_i^2F_j^2F_k^2,F_i^2F_u^2F_v^2)$
$(F_iF_jF_k,F_i^2F_j^2F_u^2)$	$(F_i^2F_jF_k,F_i^2F_u^2F_v)$	$(F_i^2F_j^2F_k,F_i^2F_k^2F_u)$	$(F_i^2F_j^2F_k^2,F_u^2F_v^2F_w^2)$

It follows from (7) that $W_u^{(i)'} W_u^{(i)} \underline{\beta}_2^{(i)}$ is a set of two equations in two elements of $\underline{\beta}_2^{(i)}$ and the checking of Rank $W_u^{(i)'} W_u^{(i)} = 2$ can be done by showing two independent unbiasedly estimable equations in elements of $\underline{\beta}_2^{(i)}$. We approach this problem by considering Table 2, the equation (5) for u = 3 and the equation (5) for the treatment with all factors at level 2.

3. Main Results

We now present our two main results and their proofs.

Theorem 1. The design $D^{(1)}$ is a search design for S1.

Proof. The proof consists of showing two independent unbiasedly estimable parametric functions of the elements in every pair in Table 3. We explain the nature of the proof by considering only one out of 21 pairs in Table 3 for the lack of space. We consider the model (4) with the elements of $\underline{\beta}_2^{(i)}$ as $(F_i F_j, F_i^2 F_j^2)$. From the design D_1 and the parametric function $S(c_u=1)$ in Table 2, it follows that the parametric functions (i)$F_i+F_i F_j$, (ii) $F_j+F_i F_j$ and (iii) F_u, $u \neq i,j$, are unbiasedly estimable. Again, from the design D_1 and $S(c_u=2)$ in Table 2, it can be seen that (iv) $F_i^2+F_i^2 F_j^2$, (v)$F_j^2+F_i^2 F_j^2$ and (vi) F_u^2, $u \neq i,j$, are unbiasedly estimable. For the treatments i,j and the other treatments in D_3, we find from the equation (5) that (vii) $\mu-F_i+F_i^2-2F_j^2-2F_i^2F_j^2$, (viii) $\mu-F_j+F_j^2-2F_i^2-2F_i^2F_j^2$ and (ix) $\mu-2(F_i^2+F_j^2)$ are unbiasedly estimable. For the

treatment with all factors at level 2, we find from (5) that (x) $\mu+F_i+F_j+F_i^2+F_j^2+F_iF_j+F_i^2F_j^2$ is unbiasedly estimable. From (ix)-(vii)+3(iv)-(i), it follows that $-F_iF_j+5F_i^2F_j^2$ is unbiasedly estimable. From (vii)+(viii)+(x)-3(ix)-6((iv)+(v)), we find that $F_iF_j-15F_i^2F_j^2$ is unbiasedly estimable. We thus have displayed two independent unbiasedly estimable parametric functions in F_iF_j and $F_i^2F_j^2$. The checkings for other pairs in Table 3 are similar to the above. This completes the proof of the theorem.

Theorem 2. The design $D^{(2)}$ is a search design for S2.

Proof. The proof consists of showing two independent unbiasedly estimable parametric functions of the elements in each of 68 pairs in Table 4 and we explain the nature of the proof by considering the only pair $(F_i^2F_jF_k, F_i^2F_j^2F_u)$. From the design D_2 and the parametric functions $S(c_v=1, c_w=1)$ it follows that (i) F_vF_w with $(v,w) \neq (j,k)$ and (ii) $F_iF_k+F_i^2F_jF_k$ are unbiasedly estimable. From the parametric functions $S(c_v=1, c_w=2)$, $S(c_v=2, c_w=1)$ and $S(c_v=2, c_w=2)$, it follows that (iii) $F_v^2F_w$ with $(v,w) \neq (i,j), (i,k), (i,u), (j,u)$, (iv) $F_i^2F_j+F_i^2F_jF_k$, (v) $F_i^2F_k+F_i^2F_jF_k$, (vi) $F_i^2F_u+F_i^2F_j^2F_u$, (vii) $F_j^2F_u+F_i^2F_j^2F_u$, (ix) $F_v^2F_w^2$ with $(v,w) \neq (i,j)$ and (x) $F_i^2F_j^2+F_i^2F_j^2F_k$ are all unbiasedly estimable. It follows from $S(c_v=1)$ and $S(c_v=2)$ that (xi) F_v, $v \neq j,k,u$, (xii) $F_j+F_i^2F_j+F_jF_k+F_i^2F_jF_k$, (xiii) $F_k+F_i^2F_k+F_jF_k+F_i^2F_jF_k$, (xiv) $F_u+F_i^2F_u+F_j^2F_u+F_i^2F_j^2F_u$, (xv) F_v^2, $v \neq i,j$, (xvi) $F_i^2+F_i^2F_j+F_i^2F_k+F_i^2F_u+F_i^2F_j^2+F_i^2F_jF_k+F_i^2F_j^2F_u$, (xvii) $F_j^2+F_j^2F_u+F_i^2F_j^2+F_i^2F_j^2F_u$ are unbiasedly estimable. For

the treatments i,j,k,u and the other (for m > 4) treatments in D_3, we get from (5), (ix), (xi) and (xv) that (xviii) $\mu+F_i^2-2F_j^2-2F_i^2F_j^2$, (xix) $\mu-F_j+F_j^2-2F_i^2+2F_i^2F_j-2F_i^2F_j^2$, (xx) $\mu-F_k-2F_i^2-2F_j^2+2F_i^2F_k+4F_i^2F_j^2$, (xxi) $\mu-F_u-2F_i^2-2F_j^2+2F_i^2F_u+2F_j^2F_u+4F_i^2F_j^2-4F_i^2F_j^2F_u$ and (xxii) $\mu-2F_i^2-2F_j^2+4F_i^2F_j^2$ are all unbiasedly estimable. (We infact will not use the equation (xxii).) From (xix) - (xviii) +3 (vii) -3 (vi) -3 (v) -6 (iv) - (ii) -3 (xvii) +3 (xvi) + (xii) and (xx) - (xviii) -9 (x) -3 (vi) -6 (v) -3 (iv) - (ii) +3 (xvi) + (xiii) we find that $-6F_i^2F_jF_k$ and $-6F_i^2F_jF_k$ $-9F_i^2F_j^2F_u$ are unbiasedly estimable. This displays two independent linear functions of $F_i^2F_j^2F_k$ and $F_i^2F_j^2F_u$ which are unbiasedly estimable. The checkings for the other pairs in Table 4 can be done similarly. This completes the proof of the theorem.

4. Concluding Remarks.

a. The design $D^{(1)}$ has (1+3m) treatments and the design $D^{(2)}$ has $(1+m+2m^2)$ treatments. Minimal resolutions III, V and VII plans require (1+2m), $(1+2m^2)$ and $(1+2m^2+8\binom{m}{3})$ treatments.

b. The design $D^{(1)}$ can search one nonzero two factor interaction. A natural question comes up, "can $D^{(1)}$ search one nonzero 3-factor or higher order interaction?" The answer is "NO". For example, in case m=5 if we consider the model (4) with the elements of $\underline{\beta}_2^{(i)}$ as $(F_1F_2, F_3^2F_4^2F_5)$, we can not find two independent unbiasedly estimable linear functions of F_1F_2 and $F_3^2F_4^2F_5$. There are in fact many such pairs.

c. By calling the level 2 as the level 0 (the method of collapsing levels), and omitting the replicated treatments, we get essentially the two series of search deisgs obtained in Srivastava and Ghosh (1976), Srivastava and Gupta (1979) for 2^m factorial experiments. However, the designs thus obtained have more strength in the sense that they can search any nonzero i-factor or higher order interactions, where i = 2 and 3.

d. This research started from an unpublished technical report of Ghosh (1985) and the motivation was to reduce the number of treatments. The designs $D^{(1)}$ and $D^{(2)}$ show the success in our effort.

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY NA	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE NA	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Technical Report No. 144	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 86-2008
6a. NAME OF PERFORMING ORGANIZATION University of California, Riverside 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
6c. ADDRESS (City, State and ZIP Code) Department of Statistics, University of California, Riverside, CA 92521	7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-86-0048
8c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 61102F PROJECT NO.: 2304 TASK NO.: A5 WORK UNIT NO.:
11. TITLE (Include Security Classification) TWO NEW SERIES OF SEARCH DESIGNS FOR 3^m FACTORIAL EXPERIMENTS	
12. PERSONAL AUTHOR(S) Subir Ghosh and Xiao Di Zhang	
13a. TYPE OF REPORT Interim 13b. TIME COVERED FROM Dec '85 TO June '86	14. DATE OF REPORT (Yr., Mo., Day) 11 July 1986 15. PAGE COUNT 15
16. SUPPLEMENTARY NOTATION Submitted to Utilitas Mathematica	

17. COSATI CODES		
FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Estimability, Interactions, Main effects, Resolution III and V plans, Search Designs.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

In this paper two new series of search designs with very small number of treatments are presented for 3^m factorial experiments. The first series of designs can search one nonzero two factor interaction and estimate it along with the general mean and the main effects. The second series can search one nonzero three factor interaction and estimate it along with the two factor and lower order interactions.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Major Brian W. Woodruff	22b. TELEPHONE NUMBER (Include Area Code) (202) 767-5027 22c. OFFICE SYMBOL AFOSR/NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

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